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DLP - Digital Line Protection

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INTRODUCTION

The DLP family of transmission line protection systems utilize wave-form sampling of the current and voltage inputs and employ appropriate algorithms and multiple microprocessors to implement the protective functions and peripheral functions such as fault location. This type of relay is commonly referred to as a "digital" or "numerical" relay.

This document provides additional detailed information about the DLP family which may not be found in the various individual instruction books. The following material covers DLP revisions A, B, and C unless explicitly stated otherwise in a particular section. Refer to the instruction book of a particular DLP relay to determine which functions described below are contained in that relay.

DIGITAL RELAY CONCEPTS -

The basic concepts of digital relay design and operation are discussed in several IEEE publications and other technical papers. The IEEE tutorial course "Microprocessor Relays and Protection Systems", publication number 88EH0269-1-PWR, is a good reference.

Sampling Rate -

The DLP relays sample current and voltage 16 times per cycle. Via a setting, The DLP relays operate at either a nominal 60 Hz or 50 Hz system frequency, and "track" frequency excursions ±5% off nominal by adjusting the sampling interval. For example, the sampling interval at 60 Hz is 1.0417 milliseconds, and it is 1.0776 milliseconds at 58 Hz.

Discrete Fourier Transform -

The DLP relays use a recursive Discrete Fourier Transform (DFT) to create phasor quantities from the sampled values of current and voltage. Prior to input into the recursive DFT, the sampled values of current are first processed to remove any dc offset that may be present. A "software transactor" algorithm is used to remove the dc offset. A "transactor" is a transformer incorporating an air gap in the magnetic core; current is applied to the primary winding and voltage is obtained from the secondary winding. Because of the air gap, the secondary voltage, V, is related to the primary current, I, by the expression:

$$V = IZ$$

where: Z = transfer impedance of the transactor

Transactors have been used extensively in electromechanical and static analog relays to effectively remove dc offset from the current. In addition, the transfer impedance, Z, establishes the base reach magnitude and angle of maximum reach (torque) for distance functions.

The transactor's transfer impedance, Z, is also known as a "replica" or "mimic" impedance. If a current is passed through an impedance that is a replica of the power system impedance, the resulting voltage across the replica impedance will not contain any dc offset that may be present in the current. This is a well known technique that is used extensively in relay designs, and the transactor is one implementation of this technique.

The general equation for fault current which includes dc offset is:

$$i(t) = I*\sin(w*t + \alpha - \theta) - I*\sin(\alpha - \theta)*e^{-t/T}$$

where: I = peak value of current
w = 2*π*f
α = angle on voltage wave at which fault occurs
θ = arctan (w*L/R) = impedance angle of power system
T = L/R (of power system)

An impedance, R2 + jw*L2, is considered to be a replica impedance if:

$$L/R = L2/R2 \quad \text{- or -}$$

$$\theta = \arctan(w*L/R) = \theta 2 = \arctan(w*L2/R2)$$

The voltage across this replica impedance is:

$$v(t) = i(t)*R2 + \frac{d}{dt} i(t)*L2$$

$$i(t)*R2 = R2*I*\sin(w*t + \alpha - \theta) - R2*I*\sin(\alpha - \theta)*e^{-t/T}$$

$$\frac{di}{dt} * L2 = w*L2*I*\cos(wt + \alpha - \theta) + L2*\frac{R}{L}*I*\sin(\alpha - \theta)*e^{-t/T}$$

The exponential part of v(t) is:

$$\left(L2*\frac{R}{L} - R2 \right)*I*\sin(\alpha - \theta)*e^{-t/T}$$

Since R/L = R2/L2,

$$\left(L2*\frac{R2}{L2} - R2 \right) = 0$$

and the dc offset is removed. The voltage is then

$$v(t) = R2*I*\sin(w*t + \alpha - \theta) + w*L2*I*\cos(w*t + \alpha - \theta)$$

Since,

$$A*\sin(w*t) + B*\cos(w*t) = (A^2 + B^2)^{\frac{1}{2}} \sin(w*t + \arctan(B/A))$$

then

$$v(t) = [(R2)^2 + (w*L2)^2]^{\frac{1}{2}} I*\sin(w*t + \alpha - \theta + \arctan(w*L2/R2))$$

$$v(t) = [(R2)^2 + (w*L2)^2]^{\frac{1}{2}} I*\sin(w*t + \alpha)$$

The conclusion is that the voltage across a replica impedance does not contain any dc offset and it is phase shifted θ degrees leading compared to the current. Appendix I shows the same results graphically using a PC mathematics software package.

In the DLP relays, all current values are processed by the "software transactor" algorithm. This algorithm uses consecutive samples of current at "t" and "t + δ" where δ is the sampling interval. The desired quantity is iZ = (R*i + L*di/dt) where, ideally, L/R in the DLP has the same value as the L/R of the transmission line. The DLP calculates L/R from the values of two settings POSANG and ZERANG. Consequently, it is important that both POSANG and ZERANG be set as close to their respective actual line angles as possible.

(R*i + L*di/dt) is approximated at time "t + $\delta/2$" which is the mid-point between two samples. The approximation is:

$$iZ = R\,\frac{i(t+\delta) + i(t)}{2} + L\,\frac{i(t+\delta) - i(t)}{\delta}$$

All currents are converted to iZ samples and properly scaled prior to being used by the recursive DFT algorithm.

The "full-cycle" DFT calculations implemented within the DLP are shown below:

$$REAL = \frac{2}{N} \sum_{k=1}^{N} s_k \sin \frac{2\pi k}{N}$$

$$IMAG = \frac{2}{N} \sum_{k=1}^{N} s_k \cos \frac{2\pi k}{N}$$

where: N = 16 samples

s_k = magnitude of kth sample of signal s(t)

The fundamental frequency (60 or 50 Hz) phasor magnitude and angle are then calculated as shown below:

$$\text{MAGNITUDE} = (\text{REAL}^2 + \text{IMAG}^2)^{\frac{1}{2}} \quad \text{(peak value)}$$

$$\text{ANGLE} = \arctan\ (\text{IMAG}/\text{REAL})$$

The DLP relays use both full-cycle and half-cycle "data windows" for the DFT calculations. The term "data window" refers to the time interval or, alternatively, the number of samples used in the DFT summation. The actual DFT algorithm is recursive. A recursive DFT uses a sliding data window where the calculations shown above are performed each time a new sample is available. The newest sample becomes k=N and what was previously sample k=1 is discarded. Such a recursive implementation maintains a constant phase angle reference so that the phase angle does not rotate but remains fixed for a steady sine wave input. Appendix II shows that the DFT calculations will produce the proper magnitude and phase angle when the waveform being sampled is a symmetrical sine wave.

By definition, the DFT calculations shown above extract the fundamental frequency component from the sampled waveform. A full-cycle data window DFT does a good but not perfect job of filtering out any dc offset as indicated on page 1 of Appendix III, but a half-cycle data window DFT does a poor job as indicated on page 2 of Appendix III. Appendix III uses the same current waveform from Appendix I. Equations (1) and (2) merely define the sampled values used by the DFT and are not part of the DFT calculations. It is the inability of the DFT to eliminate the dc offset that requires the use of the "software transactor" algorithm described previously.

HARDWARE & FIRMWARE -

The DLP relays use three microprocessors; (2) 80C186 and (1) TMS320C10. The HARDWARE DESCRIPTION and MODULES sections of each instruction book provides physical dimensions, front and rear view photographs, a module location diagram, an overall block diagram, and individual module block diagrams.

The three modules, SSP, DAP, and DSP, each contain a microprocessor and two EPROM sockets. When a firmware update is implemented the user will have to replace 1, 2, or 3 pairs of EPROMs depending upon the extent of the changes made to the software.

Anti-Aliasing Filter -

The ANI module contains the anti-aliasing filters. "Aliasing" is an error related to sampling rate which is manifested by lower frequencies appearing in the sampled signal which are not present in the input signal. Aliasing can be prevented by passing the input signal through an analog low-pass filter prior to sampling. This filter must have a cut-off frequency equal to or less than one-half of the sampling frequency, f_S. For the DLP, f_S is equal to 16 x SYSFREQ where SYSFREQ is either 50 Hz or 60 Hz. The lowest value of $f_S/2$ is 400 Hz at 50 Hz, and the anti-aliasing low-pass filter is a

three-pole butterworth design with a gain of -25 db at 400 Hz. This one filter design is used at both 60 and 50 Hz.

Analog to Digital Converter -

The DLP relays use one 12 bit analog-to-digital converter (ADC) with a multiplexed analog input system as shown in Figure 1. Since the total scan period for all the ADC inputs is approximately 65 microseconds, the worst error due to sequential sampling is 1.4° and "sample and hold" circuits are not required.

Six analog signals are sampled; (3) voltages proportional to VAG, VBG, and VCG and (3) currents proportional to IA, IB, and IC. The secondary voltages from the system PTs or CVTs are applied to auxiliary PTs in the DLP. These auxiliary PTs have a 14:1 ratio. The voltage samples are further attenuated 2:1 before they are applied to the ADC. The ADC's maximum input voltage is ±5 volts dc, and each ADC bit represents:

$$5/2^{11} = 2.442 \text{ millivolts}$$

Referred to secondary system values (i.e., voltage applied to the relay), one bit represents 2.442*2*14 = 68.4 millivolts. The quantization error, q, is:

$$q = 5*2^{-12} = 1.22 \text{ millivolts}$$

The quantization error referred to secondary system values, q_s, is:

$$q_s = 1.22*2*14 = 34.2 \text{ millivolts}$$

Thus an output of hexadecimal 001 (one bit) represents a secondary input voltage (at the terminals of the relay) between 34.2 and 102.6 millivolts (peak AC).

The secondary currents from the system CTs are applied to auxiliary CTs in the DLP. These auxiliary CTs have a resistor on their secondary to produce a voltage, labelled VIA, VIB, or VIC in Figure 1, that is proportional to current. To achieve the large dynamic range required for the current inputs while maintaining adequate resolution, the DLP uses two separate current inputs to the analog multiplexer. One input labelled "Ix16" in Figure 1 covers the range of 0-20 amps peak and the other labelled "I" covers the range of 0-320 amps peak. Normally the "Ix16" input samples are used, but when an overflow on the "Ix16" input is detected the time-equivalent sample from the "I" input is used instead. "Ix16" simply means that the "I" samples are multiplied by 16.

A secondary system current (i.e., current applied to the relay) of either 20 or 320 amps peak produces a 10 volt peak signal within the DLP. Like the voltage samples, the current samples are further attenuated 2:1 before they are applied to the ADC.

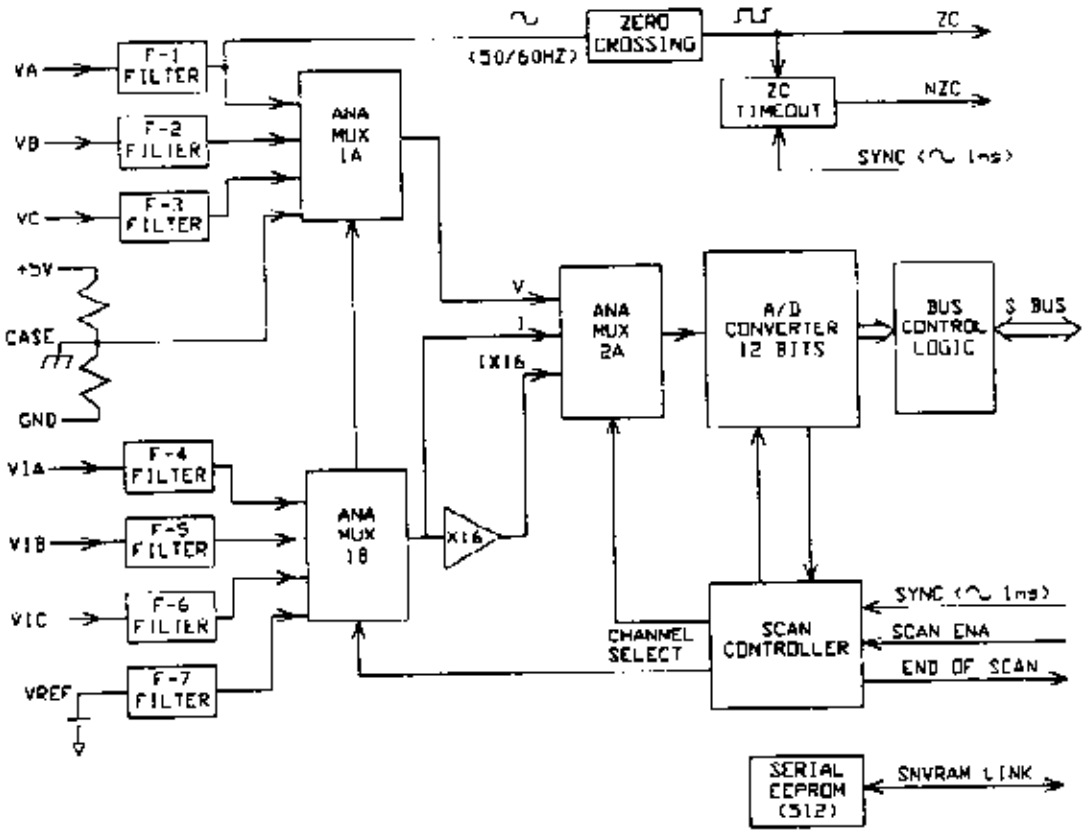


Figure 1 (Multiplexed Analog Input System)

When "Ix16" samples are used, as would be the case for normal load flow, one ADC bit is:

$$(5/2^{11})*2*(20/10) = 9.77 \text{ milliamperes (peak AC)}$$

When "I" samples are used, as would be the case for a severe fault, one ADC bit is:

$$(5/2^{11})*2*(320/10) = 156.25 \text{ milliamperes (peak AC)}$$

Considering the quantization error, one bit represents a secondary input current (at the terminals of the relay) within the ranges shown below:

"Ix16" - (4.88 - 14.65) milliamperes (peak AC)

"I" - (78.13 - 234.38) milliamperes (peak AC)

If the current applied to the relay should exceed 320 amperes peak, the resulting overflow is detected and the current value is clamped at 320 amperes.

Considering software calibration factors for the magnetics and ADC circuitry as well as software scaling factors, one bit on a voltage channel actually corresponds to 0.1 volts peak and one bit on an "Ix16" current channel actually corresponds to 0.01 amps peak.

FUNCTIONS AND FEATURES

Refer to the PRODUCT DESCRIPTION section of the instruction book for the functions and features contained in a particular DLP relay.

PHASE SELECTOR -

Versions of the DLP are available for single-phase and three-phase tripping. For single-phase tripping, the phase-selector algorithm is used to determine which phase to trip. At the heart of the phase-selector algorithm is fault type determination which is used for both single-phase and three-phase tripping to determine the fault type for the fault location algorithm. The fault component of positive-sequence current (I_{1F}), negative-sequence current (I_2), and zero-sequence current (I_0) are used to determine the fault type through a combination of level checks on I_2 and I_0 and the measurement of the phase angle between I_2 and I_{1F}. The fault component of positive-sequence current is calculated by subtracting the pre-fault current phasor from the fault current phasor.

A test level for I_2, I2LVL, and a test level for I_0, I0LVL, is fixed in the DLP's firmware. If $|I_0|$ >= I0LVL, then a ground fault is declared and the angle by which I_2 leads I_{1F} is calculated to determine which of the six ground fault types is present.

The theoretical phase angle criteria are listed below:

Ground Fault	Angle (I_2 leads I_{1F})
AG	0°
BG	120°
CG	240°
ABG	60°
BCG	180°
CAG	300°

The actual criteria allows for a +/- error about the theoretical value. The phasor relationships between I_2 and I_{1F} for all unbalanced faults are shown graphically in Figure 2. If $|I_0| <$ I0LVL, then a phase fault is declared and the angle by which I_2 leads I_{1F} is calculated to determine which of three phase-to-phase faults might be present. If none of these angle checks (same values as for phase-to-phase ground faults) are satisfied, then a three-phase fault is declared provided that $|I_2| <$ I2LVL. As a supplemental check, the fault type determined as described above is compared against "zone flags" which are generated by operation of the phase and ground distance functions. After the fault type is determined, the phase-selector function produces four outputs, phase A, phase B, phase C, and multi-phase, which cause the DLP to trip the proper phase for single-line-to-ground faults and three phase for all multi-phase faults.

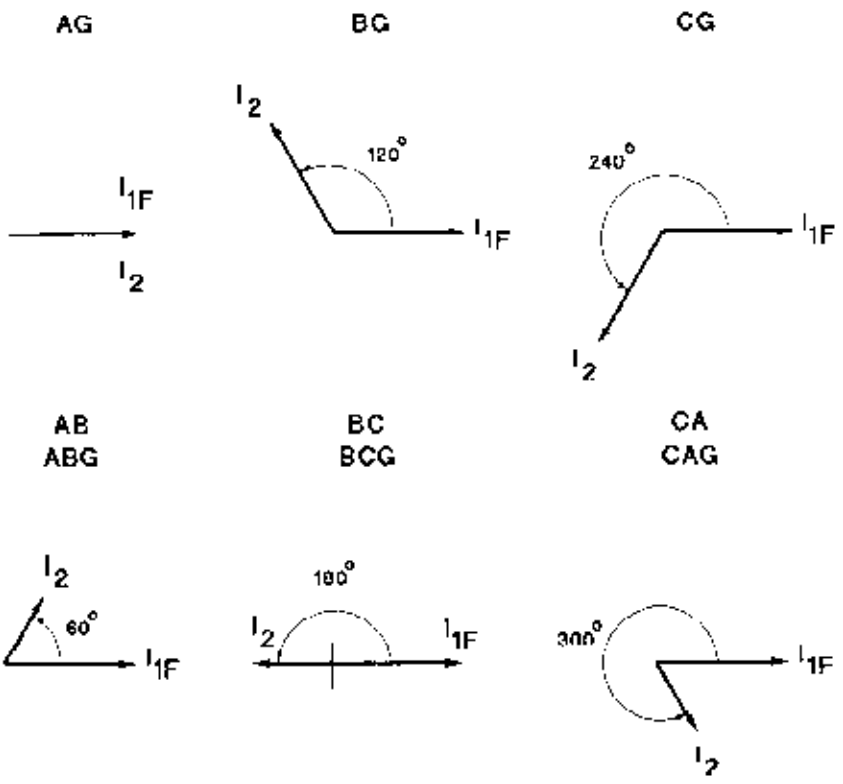


Figure 2 (Angle between I_{1F} and I_2 for Unbalanced Faults)

FAULT LOCATION -

Fault type determination is also required for fault location. Prior to running the fault location algorithm, the fault type must be known to select the proper input quantities. If fault type cannot be determined, fault location is not calculated, but the relay algorithms will still protect correctly. The fault location algorithm will be described and then the required input quantities based on fault type will be listed.

A "lumped-parameter differential equation" algorithm is used to implement a single-ended fault location estimate. "Single-ended" means that only currents and voltages available at one end of the transmission line are used. Figure 3 shows a two terminal transmission line with equivalent sources at each end. A 3-phase fault exists at n*100 percent of the line length from terminal A. Assuming the DLP is located at terminal A, the voltage at A is:

$$V_A = n*Z_L*I_A + R_f*(I_{fA} + I_{fB})$$

where: n - fraction of line length from A to the fault
Z_L = line impedance
I_A = current at A
R_f = arc resistance at the fault
I_{fA} = fault current at terminal A
I_{fB} = fault current at terminal B

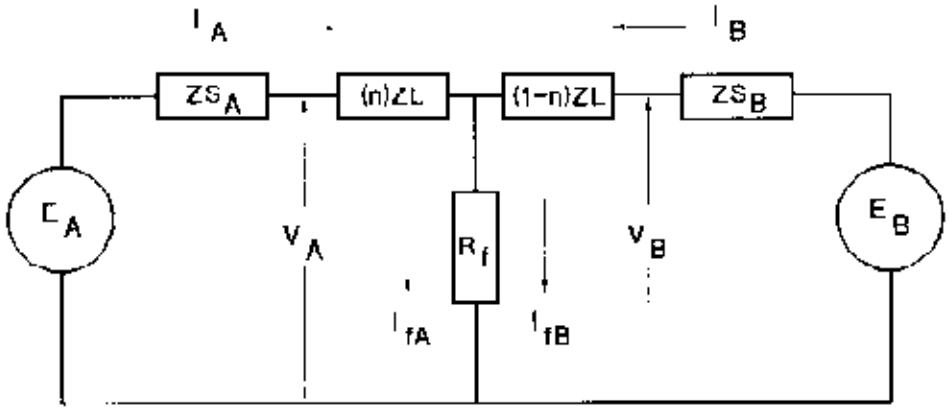


Figure 3 (Two Terminal Transmission Line)

Any pre-fault load current is removed from the total current to produce I_{fA} and I_{fB}. The term Z_L*I_A can be rewritten as:

$$Z_L*I_A = R*I_A + L*\frac{dI_A}{dt} = IZ$$

where: $Z_L = R + jw*L$ $(w = 2*\pi*f)$

The term IZ is a phasor corresponding to the sampled values iZ that are created to implement the "software transactor" algorithm as previously discussed. The iZ samples are inputs to the DFT and IZ is the phasor output.

The voltage at A can be rewritten as:

$$V_A = IZ + R_f*(I_{fA} + I_{fB})$$

A new term is defined as:

$$R_{fA}*I_{fA} = R_f*(I_{fA} + I_{fB})$$

where: $$R_{fA} = R_f*\left(1 + \frac{I_{fB}}{I_{fA}}\right)$$

to permit writing an equation for V_A in terms of quantities available at terminal A.

$$V_A = n*IZ + R_{fA}*I_{fA}$$

The term R_{fA} is a real number provided that I_{fA} and I_{fB} are in phase. For this to be true, the assumed system in Figure 3 must be homogeneous. This means that the source and line impedances must have the same angle. While this is rarely the case in an actual power system, the assumption is made here to show that the fault location algorithm eliminates the effect of fault resistance if R_{fA} is real. In practice, a typical non-homogeneous system produces only a small error in the fault location estimate.

V_A is now split into its real and imaginary components:

V_A(real) = VS[ine] V_A(imaginary) = VC[osine]

$$VS + jVC = n*(IZS + IZC) + R_{fA}*(I_{fA}S + I_{fA}C)$$

$$VS = n*IZS + R_{fA}*I_{fA}S$$
$$VC = n*IZC + R_{fA}*I_{fA}C$$

Solving for "n" yields:

$$n = \frac{VS*I_{fA}C - VC*I_{fA}S}{IZS*I_{fA}C - IZC*I_{fA}S}$$

proving that this approach eliminates the effect of fault resistance provided that the defined term R_{fA} is assumed to be real.

Like the apparent impedance seen by a distance relay, this fault location estimate is affected by current infeed from a tap or third terminal and by zero-sequence mutual impedance between parallel lines. Compensation for mutual effects using a portion of the zero-sequence current from the parallel line is not implemented in the DLP for either the fault location algorithm nor the distance functions.

As stated previously, the proper voltage, current, and IZ phasors must be used in the equation for n, and these phasors are determined by the fault type and faulted phase(s). For single-phase to ground faults, these phasors are:

a-g:
$$V = V_A$$
$$I_f = (I_A)_F$$
$$IZ = (I_A - I_0)*Z_L/\underline{POSANG} + I_0*K_0*Z_L/\underline{ZERANG}$$

b-g:
$$V = V_B$$
$$I_f = (I_B)_F$$
$$IZ = (I_B - I_0)*Z_L/\underline{POSANG} + I_0*K_0*Z_L/\underline{ZERANG}$$

c-g:
$$V = V_C$$
$$I_f = (I_C)_F$$
$$IZ = (I_C - I_0)*Z_L/\underline{POSANG} + I_0*K_0*Z_L/\underline{ZERANG}$$

For multi-phase faults these phasors are:

a-b
$$V = V_A - V_B$$
$$I_f = (I_A - I_B)_F$$
$$IZ = (I_A - I_B)*Z_L/\underline{POSANG}$$

b-c
$$V = V_B - V_C$$
$$I_f = (I_B - I_C)_F$$
$$IZ = (I_B - I_C)*Z_L/\underline{POSANG}$$

c-a
$$V = V_C - V_A$$
$$I_f = (I_C - I_A)_F$$
$$IZ = (I_C - I_A)*Z_L/\underline{POSANG}$$

where: $(\quad)_F$ indicates that the pre-fault load flow current has been subtracted from the phase currents included in $(\quad)$.

OVERCURRENT/DIRECTIONAL FUNCTIONS -

The overcurrent and directional functions contained in the DLP relays are listed below:

FD - fault detector
NT - negative-sequence directional trip function

NB - negative-sequence directional block function
IPT - ground trip overcurrent
IPB - ground block overcurrent
IDT - ground overcurrent direct trip
TOC - ground time overcurrent direct trip
IT - trip supervision overcurrent
IB - block supervision overcurrent
PH4 - non-directional phase overcurrent direct trip
Line Overload - (2) setting levels
I1 - line pickup overcurrent

Fault Detector -

The fault detector, FD, is a non-directional low-set overcurrent function with a factory set pickup level. Figure 4 is a block diagram for the FD function.

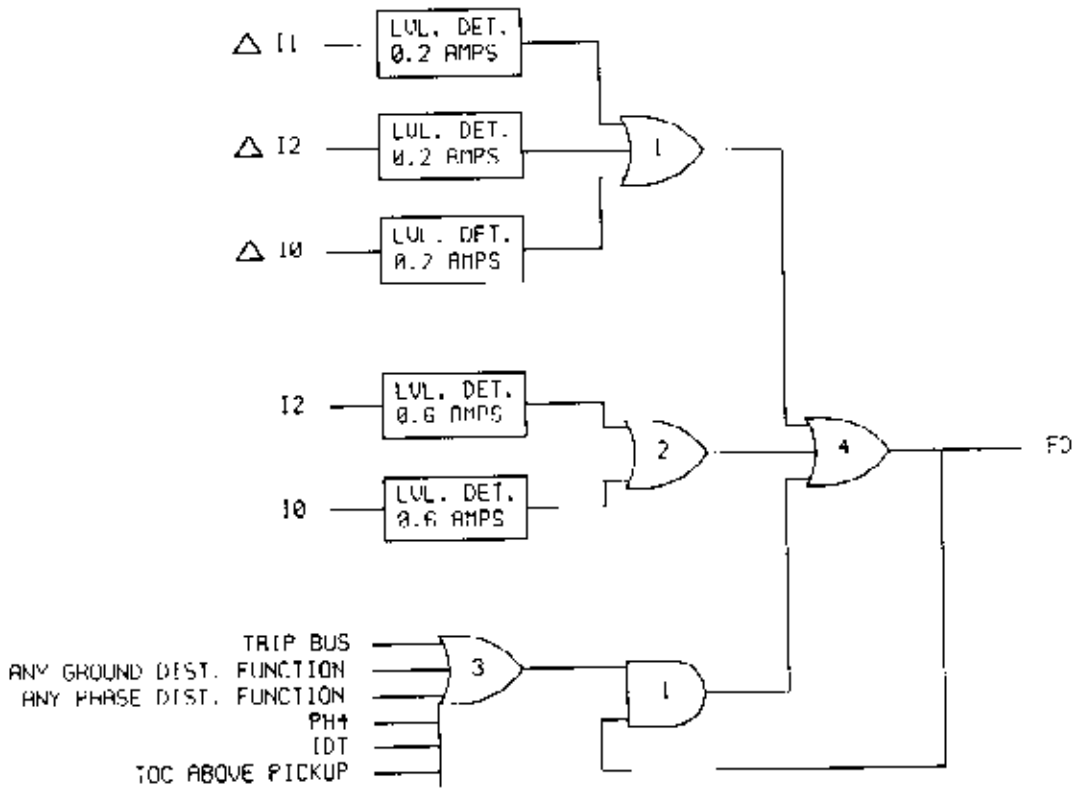


Figure 4 (Fault Detector Block Diagram)

The change ($\triangle$) in I_1, I_2, or I_0 magnitude is used together with the total magnitude of I_2 or I_0 to determine FD operation. The $|\triangle I_1|$, $|\triangle I_2|$, or $|\triangle I_0|$ quantity must exceed 0.2 amperes RMS or $|I_2|$ or $|I_0|$ must exceed 0.6 amperes RMS to produce an output. The $\triangle$ quantity magnitudes are calculated each sample by comparing the respective IZ phasor output from the DFT with a memory value of IZ stored four cycles previously. Recall that all phase-current samples are converted to iZ samples to remove the dc offset and then fed to the DFT algorithm which produces IZ phasors. These phase-current IZ phasors are then converted to IZ_1, IZ_2, and IZ_0 symmetrical component phasors.

As an example, the positive-sequence IZ_1 magnitude calculated at the latest sample is compared to the magnitude of IZ_1 stored in memory four cycles previously, and if the absolute difference exceeds 0.2 amperes RMS, FD will produce an output. The four cycle "look back" comparison used for the /\ quantities results in a four cycle "reset" time for the FD function. Since it is not only necessary that FD detect any system disturbance but also maintain a constant output during the disturbance, several internal DLP signals or flags are used to seal-in the FD output as shown in Figure 4. This level of detail is not shown on the functional logic diagrams in the instruction books.

Ground Directional Functions -

Figure 5 is a block diagram for the ground overcurrent and directional functions. Two negative-sequence directional functions, NT (trip) and NB (block), are used. It is well documented in the relay literature that negative-sequence directional functions are superior to zero-sequence current and/or voltage polarized directional functions particularly when zero-sequence mutual coupling is present between parallel lines. NT supervises the IPT pilot-trip function and provides selectable directional control (i.e., torque control) of the direct-trip backup functions, IDT (50) and TOC (51). NB supervises the IPB pilot-block function.

NT and NB are implemented using an "amplitude comparator" measurement. An amplitude comparator is a generic type of relay function measurement technique that compares the magnitude of an OPERATE quantity versus the magnitude of a RESTRAINT quantity to determine if the relay function should operate or not. If the OPERATE quantity is larger than the RESTRAINT quantity, then the relay function operates. For NT, the simplest amplitude comparator design is:

$$\text{OPERATE} = |V_2 - I_2 * Z_R|$$

$$\text{RESTRAINT} = |V_2 + I_2 * Z_R|$$

where: I_2 = negative-sequence current at relay
V_2 = negative-sequence voltage at relay
Z_R = relay "reach" impedance = $|Z_R|$ /POSANG

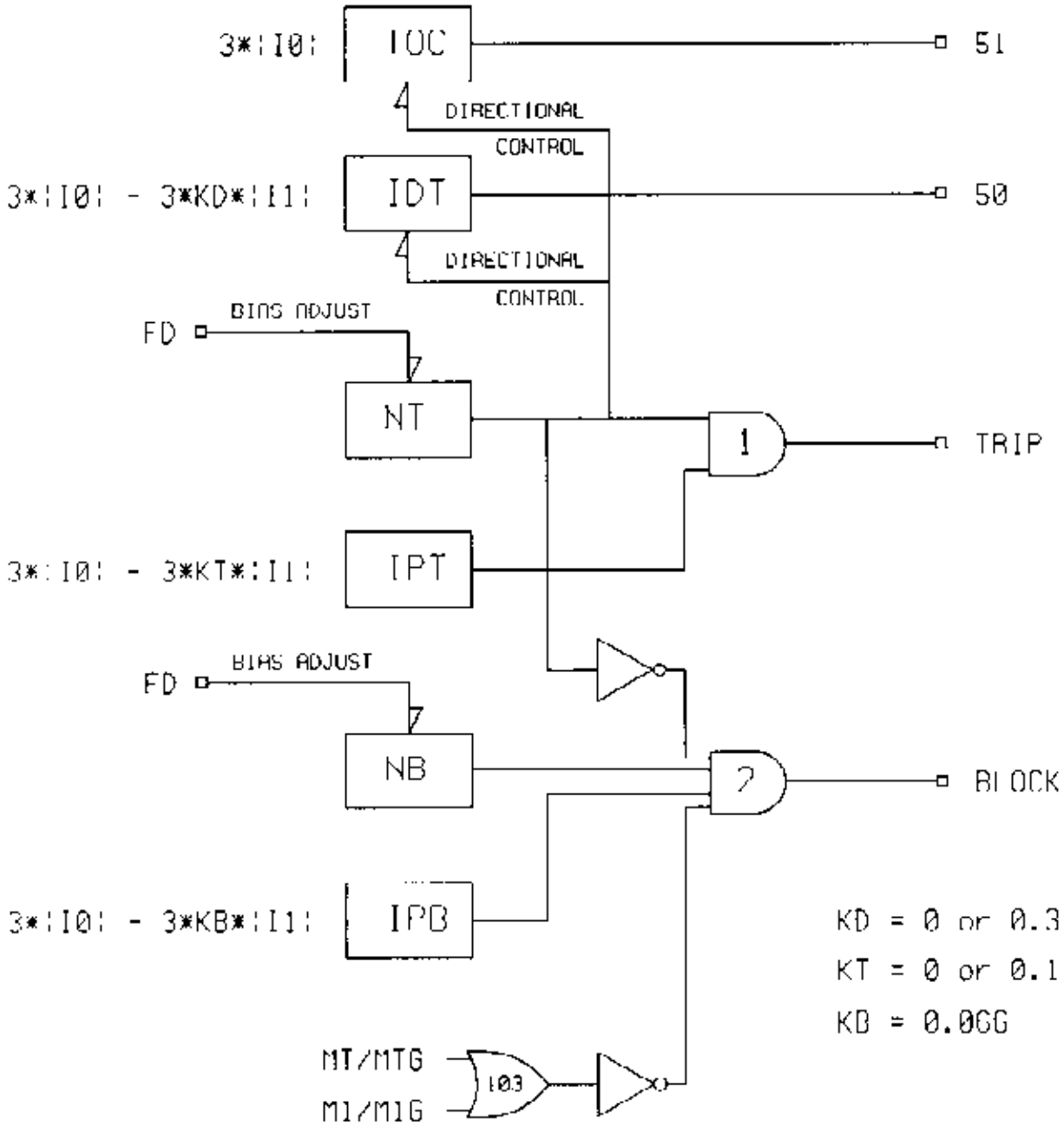


Figure 5 (Ground Directional and Overcurrent Functions)

However, for some system conditions, the negative-sequence voltage at the relay may approach zero. With V_2 approaching zero, NT would be near it's balance point, and would not operate reliably for an unbalanced fault in the forward direction. Consequently, a compensation technique is used which creates a reliable V_2 polarizing quantity in the relay even though V_2 at the relay may be zero. This compensation modifies the OPERATE and RESTRAINT quantities as shown below:

$$\text{OPERATE} = |V_2 - (1+K)*I_2*Z_R|$$

$$\text{RESTRAINT} = |V_2 + (1-K)*I_2*Z_R|$$

where: $K = 0.05$ = offset compensation

For NB, the OPERATE and RESTRAINT quantities are simply interchanged.

The actual DLP OPERATE and RESTRAINT quantities are tabulated below:

	NT	NB
OPERATE	$\|V_2 - 1.05*I_2*Z_R\|$	$\|2*V_2 + 2*I_2*Z_R\|$
RESTRAINT	$\|V_2 + I_2*Z_R\|$	$\|2.05*I_2*Z_R\|$

where: $Z_R = 20 \angle \text{POSANG}$ for a 5 ampere DLP
$Z_R = 100 \angle \text{POSANG}$ for a 1 ampere DLP

For NT to operate, OPERATE must exceed RESTRAINT by 1.0 secondary volts RMS. For NB to operate, OPERATE must exceed RESTRAINT by 0.5 secondary volts RMS. The NT RESTRAINT quantity is $|V_2 + I_2*Z_R|$ rather than the theoretical $|V_2 - (1-K)*I_2*Z_R|$. The elimination of the (1-K) factor creates a larger magnitude restraint quantity for increased security. For the NB function, both the OPERATE and RESTRAINT quantities are essentially doubled in magnitude compared to the corresponding NT values. A blocking function at one end of a protected line should operate as fast as or faster than the tripping function at the other end for a through fault condition. Since the DLP employs an enhanced amplitude comparator called an "energy comparator", doubling the NB quantities compared to NT assures more energy in NB and consequently a faster operate time compared to NT at the other end.

The basic advantage of the energy comparator is that an integrating algorithm replaces the magnitude comparison of the simple amplitude comparator. If the difference between the OPERATE and RESTRAINT signal is positive, then the integrating algorithm integrates "up" toward the operate threshold level. If this difference is negative, then the integrating algorithm integrates "down" away from the operate threshold level. This allows for modifying the response of NT and NB by adding restraint "energy" or bias when the fault detector, FD, is not operated. This is indicated in Figure 5 by the BIAS ADJUST inputs to NT and NB. When

FD operates indicating a system fault or disturbance, this bias energy is removed, and should NT or NB begin to integrate "up" toward operation the integration must overcome the initial restraint energy. Simply stated, at the instance a fault occurs both NT and NB are biased toward non-operation to increase security.

Other than the factor of two mentioned above there are other differences in the NB OPERATE and RESTRAINT quantities compared to the theoretical quantities. The RESTRAINT quantity is $|2.05*I_2*Z_R|$ rather than $|2*V_2 - 2*(1+K)*I_2*Z_R|$. Elimination of $2*V_2$ creates a smaller RESTRAINT quantity which increases dependability and speed. The OPERATE quantity is $|2*V_2 + 2*I_2*Z_R|$ rather than $|2*V_2 + 2*(1-K)*I_2*Z_R|$. Elimination of the (1-K) factor creates a larger OPERATE quantity which increases dependability and speed.

Ground Pilot Overcurrent Functions -

IPT ANDed with NT is the ground directional-overcurrent pilot trip function, and IPB ANDed with NB is the ground directional overcurrent pilot block function. These functions are labelled TRIP and BLOCK in Figure 5, and they are only in service if SELZGU = 0 or 2 and one of the pilot schemes (i.e., POTT, PUTT, BLOCK, or HYBRID) has been selected. The IPT and IPB operating quantities are:

IPT: $3*|I_0| - 3*KT*|I_1|$

IPB: $3*|I_0| - 3*KB*|I_1|$

where: KT = 0 or 0.1
KB = 0.066

For the BLOCK and HYBRID schemes, KT = 0.1. For the POTT and PUTT schemes, KT = 0. The KT and KB constants are not directly user selectable but are set to the appropriate value based on the SELSCHM setting. Figure 5 shows that the local NT, zone 1 distance, and zone 2 distance functions (via OR 103) block NB*IPB at AND 2. This local "trip over block" preference is part of the transient blocking logic and is not shown on the functional logic diagrams in the instruction books.

IPT and IPB utilize positive-sequence current restraint, $3*KB*|I_1|$, to limit their reach for external faults and to prevent possible operation due to system or load dissymmetry under maximum load flow conditions. This feature increases scheme security when a BLOCKING or HYBRID scheme is selected. When a POTT or PUTT tripping scheme is selected, the positive-sequence current restraint is eliminated from the NT function (KT = 0) to assure that it operates for all remote terminal faults under the most severe system and load conditions.

Ground Backup Overcurrent Functions -

An instantaneous (IDT) and time overcurrent (TOC) function provide direct-trip backup protection for ground faults. The IDT

operating quantity is:

$$3*|I_0| - 3*KD*|I_1|$$

where: KD = 0 or 0.3

In some DLP relays, KD is fixed at 0.3. In other DLP relays, KD can be set to 0 or 0.3. The setting determination is more difficult (see the instruction books) when KD = 0.3, but overreach possibilities are greatly reduced while sensitivity is increased after the remote breaker trips.

The TOC operating quantity is simply $3*|I_0|$. Depending on the version of the DLP relay, the characteristic curve shape is either fixed or user selectable. Both IDT and TOC can be separately controlled by the NT directional function or operated as non-directional functions. This "directional control" is analogous to "torque control" in EM relays since the IDT or TOC algorithm does not begin executing until NT operates.

Phase-Current Actuated Overcurrent Functions -

IT (trip) and IB (block) are two low-set overcurrent supervision functions each of which have seven outputs. As shown in Figure 6, three outputs (ITA, ITB, ITC) are per phase, three outputs (ITAB, ITBC, ITCA) are per phase-pair, and one (IT) is a logical OR of the per phase outputs. The settings, PUIT and PUIB, are in terms of RMS secondary phase current. As shown in Figure 6, the threshold level for the per phase-pair outputs is $\sqrt{3}$*PUIT.

IB provides overcurrent supervision of the zone 4 distance functions. IT provides overcurrent supervision of all other distance functions (refer to the next section). Ground distance functions are supervised by ITA, ITB, or ITC. Phase distance functions are supervised by ITAB, ITBC, or ITCA. IT and IB are also used for other purposes as shown on the functional logic diagrams in the instruction books. Supervision of the distance functions by these two overcurrent functions is primarily intended to provide a fast reset of the distance functions; it <u>does not</u> provide blown fuse protection since PUIT and PUIB are normally set well below the full-load current. A separate function is provided in the DLP relays to detect a blown AC potential fuse.

PH4 is an instantaneous non-directional overcurrent direct trip function. It is intended to provide direct tripping for multi-phase faults, and it operates on the highest of the three delta currents, $I_A - I_B$, $I_B - I_C$, or $I_C - I_A$. The pickup setting, PUPH4, is based on the delta current magnitude not the phase current magnitude.

There are two overcurrent functions (two settings - PULV1 and PULV2) associated with the Line Overload function. Each function operates on the highest of the three phase currents, I_A, I_B, or I_C.

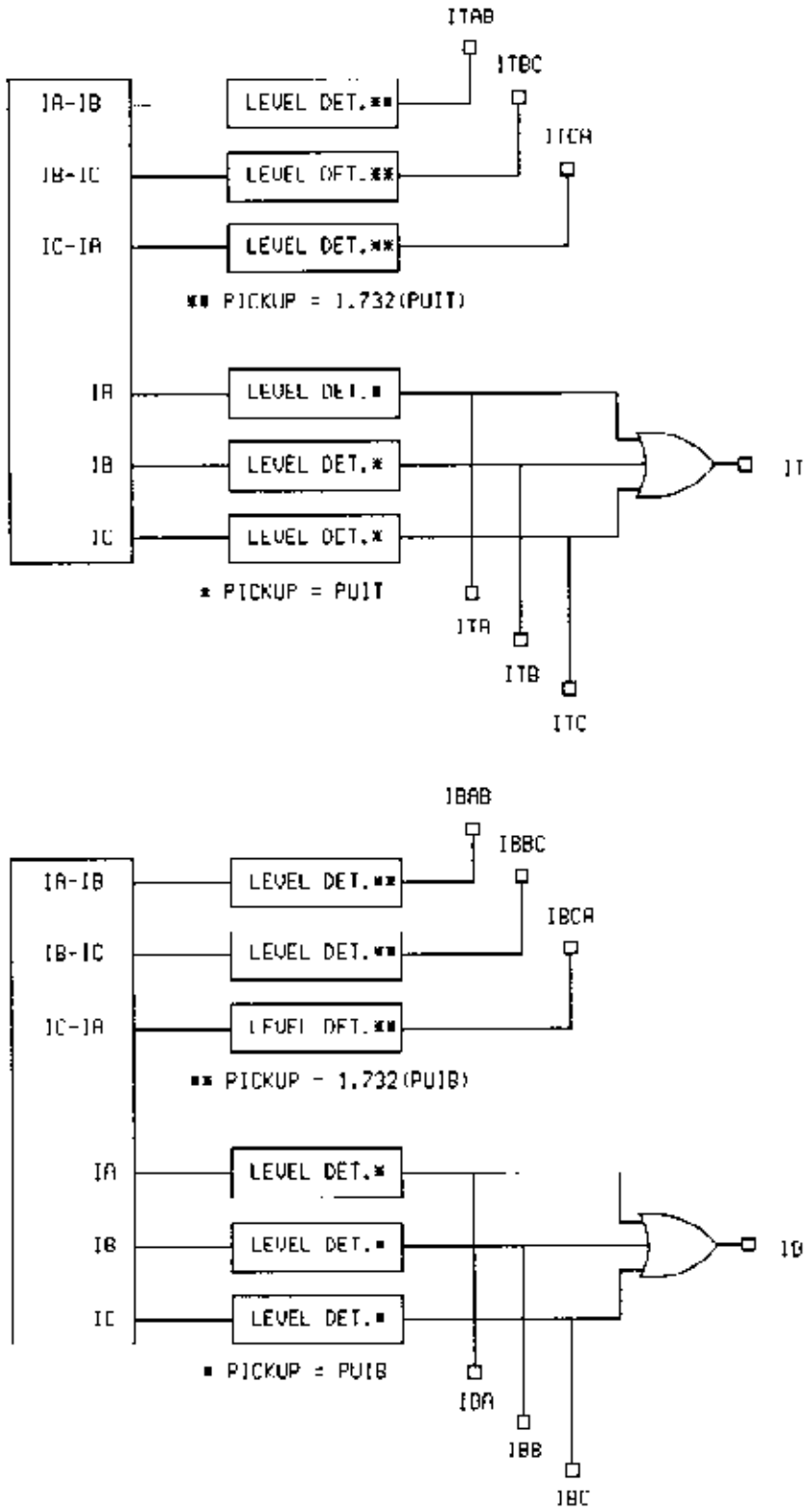


Figure 6 (Overcurrent Supervision Functions)

Line Pickup Overcurrent Function -

The overcurrent function, I1, associated with Line Pickup operates on the magnitude of positive-sequence current. Operation of Line Pickup is described in the instruction books.

DISTANCE FUNCTIONS -

In this section, the phasor inputs to the Phase Angle Comparator for each distance function are listed. This information will allow the user to analytically determine the steady-state response of any distance function for any fault. An example of such an analysis is given at the end of this section.

A Phase Angle Comparator is a measurement technique that determines the amount of coincidence between two or more phasor inputs (in electrical degrees) to determine if the relay function should operate or not. References 1, 2, and 3 provide more detailed information about Phase Angle Comparators. In the DLP, the associated phasor inputs (magnitude and angle) are first calculated and stored in memory then their electrical coincidence is determined using a simple software routine.

For each zone of distance protection there are six (6) separate distance functions. There are three phase distance functions - one per phase pair (i.e., AB, BC, and CA). There are three ground distance functions - one per phase (i.e., A, B, and C).

The following definitions pertain to all of the distance functions:

I_A = phase A current at relay
I_B = phase B current at relay
I_C = phase C current at relay
I_0 = zero-sequence current at relay
V_A = phase A to ground voltage at relay
V_B = phase B to ground voltage at relay
V_C = phase C to ground voltage at relay
$(\quad)_1$ = positive-sequence component of ( )
$(\quad)_2$ = negative-sequence component of ( )
$(\quad)_M$ = memory (pre-fault) value of ( )
Zx1 = zone x pos.-seq. reach setting - ZxR /POSANG
Zx0 = zone x zero-seq. reach setting - ZxR /ZERANG

[note: the magnitudes of Zx1 and Zx0 are identical - these quantities differ in phase angle only]

Z4OR = zone 4 offset reach multiplier
K01 = zone 1 zero-sequence compensation factor - Z1K0
K0 = zero-sequence compensation factor

In the DLP relays, K0 is equal to Z_0 of the line divided by Z_1 of the line.

Zone 1 Distance Functions -

The variable MHO zone 1 distance Phase Angle Comparators are fixed at 90° coincidence to produce a circular characteristic.

Phase Distance - variable MHO:

AB: $(I_A - I_B)*Z11 - (V_A - V_B)$
$(V_A - V_B)_{1M}$
$(I_A - I_B)*Z11$

BC: $(I_B - I_C)*Z11 - (V_B - V_C)$
$(V_B - V_C)_{1M}$
$(I_B - I_C)*Z11$

CA: $(I_C - I_A)*Z11 - (V_C - V_A)$
$(V_C - V_A)_{1M}$
$(I_C - I_A)*Z11$

Ground Distance - variable MHO:

A: $(I_A - I_0)*Z11 + K01*I_0*Z10 - V_A$
$(V_A)_{1M}$
$(I_A)_2*Z11$
I_0*Z11

B: $(I_B - I_0)*Z11 + K01*I_0*Z10 - V_B$
$(V_B)_{1M}$
$(I_B)_2*Z11$
I_0*Z11

C: $(I_C - I_0)*Z11 + K01*I_0*Z10 - V_C$
$(V_C)_{1M}$
$(I_C)_2*Z11$
I_0*Z11

Ground Distance - reactance:

A: $(I_A - I_0)*Z11 + K01*I_0*Z10 - V_A$
$(I_A)_2*Z11$
I_0*Z11

B: $(I_B - I_0)*Z11 + K01*I_0*Z10 - V_B$
$(I_B)_2*Z11$
I_0*Z11

C: $(I_C - I_0)*Z11 + K01*I_0*Z10 - V_C$
$(I_C)_2*Z11$
I_0*Z11

The supervising MHO that is part of the total reactance function has the same phasor inputs as the zone 1 ground MHO unit except that Z11 is determined by the setting Z1SU. The Z1SU reach adapts to load flow conditions. Reference the CALCULATION OF SETTINGS section in the instruction books.

Zone 2 Distance Functions -

The number of electrical degrees of coincidence required to produce an output from the Phase Angle Comparators of the variable MHO zone 2 distance functions is determined by the settings Z2PANG and Z2GANG.

Phase Distance - variable MHO:

AB: $(I_A - I_B)*Z21 - (V_A - V_B)$
$(V_A - V_B)_{1M}$
$(I_A - I_B)*Z21$

BC: $(I_B - I_C)*Z21 - (V_B - V_C)$
$(V_B - V_C)_{1M}$
$(I_B - I_C)*Z21$

CA: $(I_C - I_A)*Z21 - (V_C - V_A)$
$(V_C - V_A)_{1M}$
$(I_C - I_A)*Z21$

Ground Distance - variable MHO:

A: $(I_A - I_0)*Z21 + K0*I_0*Z20 - V_A$
$(V_A)_{1M}$
$(I_A)_2*Z21$
I_0*Z21

B: $(I_B - I_0)*Z21 + K0*I_0*Z20 - V_B$
$(V_B)_{1M}$
$(I_B)_2*Z21$
I_0*Z21

C: $(I_C - I_0)*Z21 + K0*I_0*Z20 - V_C$
$(V_C)_{1M}$
$(I_C)_2*Z21$
I_0*Z21

Zone 3 Distance Functions -

The number of electrical degrees of coincidence required to produce an output from the Phase Angle Comparators of the variable MHO zone 3 distance functions is determined by the settings Z3PANG and Z3GANG.

Phase Distance - variable MHO:

AB: $(I_A - I_B)*Z31 - (V_A - V_B)$
$(V_A - V_B)_{1M}$
$(I_A - I_B)*Z31$

BC: $(I_B - I_C)*Z31 - (V_B - V_C)$
$(V_B - V_C)_{1M}$
$(I_B - I_C)*Z31$

CA: $(I_C - I_A)*Z31 - (V_C - V_A)$
$(V_C - V_A)_{1M}$
$(I_C - I_A)*Z31$

Ground Distance - variable MHO:

A: $(I_A - I_0)*Z31 + K0*I_0*Z30 \quad V_A$
$(V_A)_{1M}$
$(I_A)_2*Z31$
I_0*Z31

B: $(I_B - I_0)*Z31 + K0*I_0*Z30 \quad V_B$
$(V_B)_{1M}$
$(I_B)_2*Z31$
I_0*Z31

C: $(I_C - I_0)*Z31 + K0*I_0*Z30 - V_C$
$(V_C)_{1M}$
$(I_C)_2*Z31$
I_0*Z31

Zone 4 Distance Functions -

The number of electrical degrees of coincidence required to produce an output from the Phase Angle Comparators of the variable MHO zone 4 distance functions is determined by the settings Z4PANG and Z4GANG.

Phase Distance - variable MHO:

AB: $(I_A - I_B)*Z41 - (V_A - V_B)$
$(I_A - I_B)_1*Z4OR*Z41 - (V_A - V_B)_{1M}$
$(I_A - I_B)*Z41$

BC: $(I_B - I_C)*Z41 \quad (V_B - V_C)$
$(I_B - I_C)_1*Z4OR*Z41 - (V_A - V_B)_{1M}$
$(I_B - I_C)*Z41$

CA: $(I_C - I_A)*Z41 - (V_C - V_A)$
$(I_C - I_A)_1*Z4OR*Z41 - (V_A - V_B)_{1M}$
$(I_C - I_A)*Z41$

The above phasor inputs are for the forward reach case where SELZ4D = 0. For the reverse reach case, where SELZ4D = 1, simply put a minus sign in front of each current expression.

Ground Distance - variable MHO:

A: $(I_A - I_0)*Z41 + K0*I_0*Z40 - V_A$
$(V_A)_{1M}$
$(I_A)_2*Z41$
I_0*Z41

B: $(I_B - I_0)*Z41 + K0*I_0*Z40 - V_B$
$(V_B)_{1M}$
$(I_B)_2*Z41$
I_0*Z41

C: $(I_C - I_0)*Z41 + K0*I_0*Z40 - V_C$
$(V_C)_{1M}$
$(I_C)_2*Z41$
I_0*Z41

The above phasor inputs are for the forward reach case where SELZ4D = 0. For the reverse reach case, where SELZ4D = 1, simply put a minus sign in front of each current expression.

Analysis Using Phasor Input Coincidence -

The following is an example of how calculation of phasor input coincidence can be used to determine if a distance function should or should not operate for a given fault condition. For the sample transmission system of Figure CS-1 in the instruction books, assume a DLP relay is located at terminal Able with a bolted A-G fault located at terminal Delta. The phase A, zone 3, ground MHO function will be analyzed. The instruction book calculation shows that the apparent impedance seen at terminal Able for this fault is 16 $\angle 80.3°$ secondary ohms assuming no pre-fault load flow (i.e., voltages E1, E2, and E3 are all at 0°). For this condition, the phase A, zone 3, ground MHO function is at a virtual balance point with Z3GR (Z31) = 16.00. The evaluation below takes into account pre-fault load flow (i.e., $\angle E1$ = 0°, $\angle E2$ = -35°, $\angle E3$ = -30°).

Z3GR (Z31) = 16.00
POSANG = 85°
ZERANG = 74°
K0 = 3.2
Z3GANG = 90

The fault currents and voltages at the relay are listed below:

pre-fault $V_A = 64.87 \angle -5.6°$
$V_A = 59.93 \angle -3.9°$
$(V_A)_1 = 62.82 \angle -4.9°$
$I_A = 3.915 \angle -53.9°$
$(I_A)_2 = 1.099 \angle -111.2°$
$I_0 = 0.493 \angle -105.1°$

The four phasor inputs are calculated to be:

1) $(I_A - I_0)*Z31 + K0*I_0*Z30 - V_A = 27.26 \angle 72.8°$
2) $(V_A)_{1M} = 64.87 \angle -5.6°$
3) $(I_A)_2*Z31 = 17.58 \angle -26.2°$
4) $I_0*Z31 = 7.89 \angle -20.1°$

The number of electrical degrees of coincidence, considering all four phasors simultaneously, must be determined. In this case, all four phasor inputs must overlap for at least 90°. This overlap or coincidence is equal to:

$$180° + (-26.2°) - 72.8° = 81°$$

Since 90° of coincidence is required for operation (Z3GANG = 90), the phase A, zone 3, ground MHO function will not operate for this fault condition.

This method of determining distance function operation is preferable to plotting impedances on an R-X diagram since the variable MHO characteristic changes its size and position on the R-X diagram depending on pre-fault load flow, system impedances, and fault type. In this example, a "no pre-fault load flow" calculation indicates that a 16.00 ohm reach setting is just barely adequate. However, when pre-fault load flow is taken into account the 16.00 ohm reach is not adequate. In fact, the reach must be increased to approximately 18.00 ohms to assure operation for this condition.

Current Sensitivity -

Any distance function requires a minimum value of current to assure operation. In the instruction books for the DLP relays, formulas for determining distance function current sensitivity are located near the end of the CALCULATION OF SETTINGS tab section. These formulas are based on the inherent sensitivity of the distance functions and do not take into account the pickup settings of the IT or IB overcurrent supervision functions.

There is no one value of fault (or test) current below which a distance function will not operate and above which it will reach 100% of its set reach. Instead, the actual reach decreases compared to the set reach as the current decreases. For electromechanical distance relays reach versus current was plotted as a "bullet curve." This decrease in reach is often expressed as "pull-back." For instance, if the set reach is 4 ohms but the actual reach at a certain level of current is 3.6 ohms, then the reach "pull-back" is 0.4 ohms or 10% of the set reach. The current sensitivity formulas in the DLP instruction books utilize this concept of reach "pull-back."

Polarizing Voltage Sensitivity -

The MHO distance functions utilize positive-sequence voltage with pre-fault memory for the polarizing quantity. This means that for any bolted zero-voltage fault, other than a three-phase fault, there will be sufficient polarizing voltage to keep the distance functions operated as long as the fault persists (assuming sufficient fault current). For a bolted three-phase fault, the phase distance functions will operate transiently because of the pre-fault memory voltage but will eventually reset if the memory voltage resets before the fault is cleared.

The minimum value of positive-sequence polarizing voltage required at the DLP to keep the MHO distance functions (all zones) operated for as long as a three-phase fault persists is 5% of 67 volts or 3.35 volts RMS. This fact is important when providing zone 2 backup protection on a "short" line. Depending on the source to line ratio, the voltage at the relay may approach zero for a three-phase fault at the remote terminal. Unless the relay voltage is 3.35 volts or greater, the zone 2 function will not stay picked up long enough to time out the zone 2 timer.

It should be noted that some level of polarizing voltage sensitivity is desirable. For a three-phase fault with arc resistance directly behind the relay, the effect of pre-fault load flow in the non-tripping direction causes the arc voltage to shift in phase towards the operating region of the distance function. The effect of polarizing voltage sensitivity is to raise the threshold of the polarizing voltage above that produced by the arc drop and so prevent operation.

Operating Time Curves -

One of the phasor inputs listed above that is common to all of the distance functions is "IZ-V" which is often termed the operating quantity. IZ-V has two basic forms that differ between phase and ground distance functions as shown below.

	IZ-V
phase variable MHO:	$I_{\phi\phi}*Z_{R1} - V_{\phi\phi}$
ground variable MHO:	$(I_{\phi}-I_0)*Z_{R1} + K0*I_0*Z_{R0} - V_{\phi}$

where:
$I_{\phi\phi}$ = phase-to-phase current at relay (e.g., $I_A - I_B$)
I_{ϕ} = phase current at relay (e.g., I_A)
I_0 = zero-sequence current at relay
$V_{\phi\phi}$ = phase-to-phase voltage at relay (e.g., $V_A - V_B$)
V_{ϕ} = phase-to-ground voltage at relay (e.g., V_A)
Z_{R1} = positive-sequence reach setting - $|Z_{R1}|$ /POSANG
Z_{R0} = zero-sequence reach setting - $|Z_{R1}|$ /ZERANG
K0 = zero-sequence compensation factor

The operating times of the MHO distance functions are directly related to the magnitude of the operating quantity. Figures 7 and 8 plot operating time versus the magnitude of IZ-V. **These two figures are only applicable to DLP revision C relays.** The inverse nature of these time versus IZ-V curves results from an intentional design effort to optimize speed versus security. Simply stated, a heavy close-in fault (high IZ-V) has the fastest tripping time because such a fault poses the greatest threat to power system security. Power system security is impacted less for remote line-end faults (low IZ-V), but relay security is more critical because the fault is closer to the balance point of the distance function. It is therefore prudent to increase the tripping time for low values of IZ-V to assure relay security without sacrificing power system

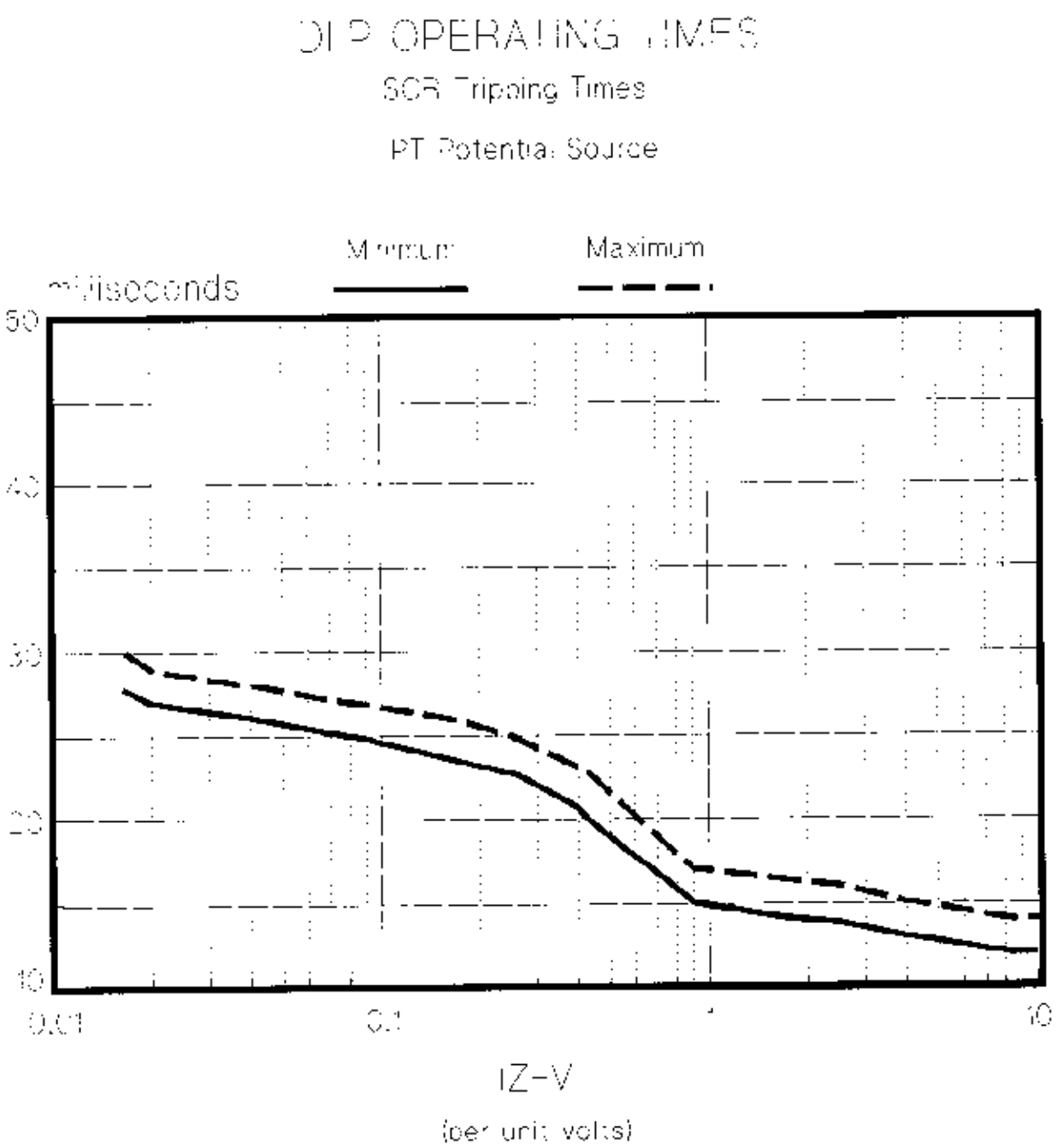


Figure 7 (Operating Times with PT Potential Sources)

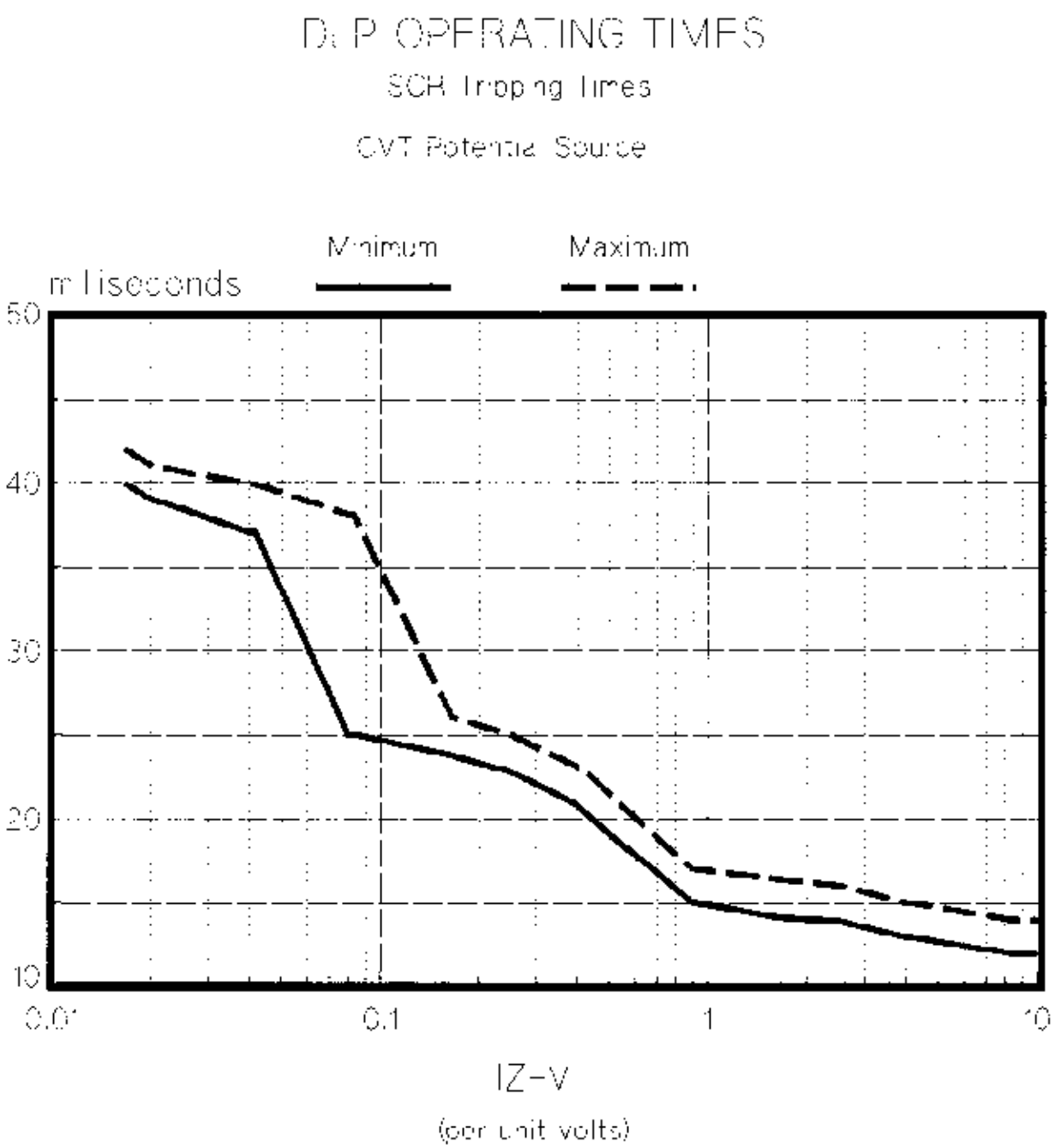


Figure 8 (Operating Times with CVT Potential Sources)

security. Stated another way, power system security could be severely compromised if a zone 1 distance function were to over-reach for a remote bus fault.

For the DLP revision C relays, the setting SELPRIM can be set to either 0(CVT PRI), 1(CVT SEC), 2(PT PRI), or 3(PT SEC). This one setting determines two different aspects of the DLP's operation. First, the setting determines how PRESENT VALUES are displayed. With SELPRIM = 0 or 2, the PRESENT VALUES (currents, voltages, watts, and vars) are displayed and stored as primary values. With SELPRIM = 1 or 3, the PRESENT VALUES are displayed and stored as secondary values. All settings are expressed in terms of secondary values, regardless of the SELPRIM setting.

Secondly, the setting determines the amount of filtering used in the DLP's distance functions to overcome transient error signals that may be present in the AC voltages. When magnetic voltage transformers (PTs) are used, SELPRIM = 2 or 3 should be selected, depending upon whether PRESENT VALUES are to be displayed in primary or secondary values. When capacitive voltage transformers (CVTs) are used, SELPRIM = 0 or 1 should be selected, depending upon whether PRESENT VALUES are to be displayed in primary or secondary values. With SELPRIM = 0 or 1 the operating time of the distance functions will be slower at lower values of operating signal (IZ-V) where the transient error signals associated with CVTs become significant. Note that when CVTs are used, a setting of SELPRIM = 2 or 3 may result in zone 1 overreach for line-end faults. Therefore SELPRIM must be set to 0 or 1 when CVTs are used.

Figure 7 applies when SELPRIM = 2 or 3, and Figure 8 applies when SELPRIM = 0 or 1. Note that IZ-V is expressed as a per unit value. The per unit "base" is 67 volts for a ground distance function and $\sqrt{3}*67 = 116$ volts for a phase distance function. To use these curves, the appropriate currents and voltages at the relay must be determined from a short-circuit analysis and the DLP relay settings must be selected for the particular zone of protection being analyzed. Figures 7 and 8 are valid for all the variable MHO distance functions regardless of the zone (i.e., zone 1, zone 2, zone 3, or zone 4).

The following is an example of how to calculate operating time using Figures 7 and 8. For the sample transmission system of Figure CS-1 in the instruction books, the operating time of the zone 1 ground and phase distance MHO functions will be computed for a DLP revision C relay located at terminal Able. The zone 1 reach is 90% of the line A-B positive-sequence impedance and the faults are located at 60% of the line A-B positive-sequence impedance in front of the relay.

The relay settings are:

Z1R = 5.40
Z1GR = 5.40

Z1K0 = 3.0
POSANG = 85
ZERANG = 74

Therefore:

$Z_{R1} = 5.40 \angle 85°$
$Z_{R0} = 5.40 \angle 74°$

For an a-g fault:

$I_A = 8.686 \angle -73.4°$
$I_0 = 2.696 \angle -95.7°$
$V_A = 50.06 \angle -3.9°$

$$IZ\text{-}V = (I_A - I_0) * Z_{R1} + K0 * I_0 * Z_{R0} - V_A = 22.3 \angle -1.63°$$

$$IZ\text{-}V \text{ (per unit)} = \frac{22.3}{67} = 0.333$$

From Figure 7, the operating time is 21.5 to 24 milliseconds.

For a B-C fault:

$I_B - I_C = 20.53 \angle -175°$
$V_B - V_C = 73.94 \angle -90°$

$$IZ\text{-}V = (I_B - I_C) * Z_{R1} - (V_B - V_C) = 36.97 \angle -90°$$

$$IZ\text{-}V \text{ (per unit)} = \frac{36.97}{116} = 0.319$$

From Figure 7, the operating time is 22 to 24.5 milliseconds.

For this example, Figure 7 or Figure 8 will produce the same operating times since the curves overlap when IZ-V is greater than approximately 0.2 per unit. Note that the operating times from Figures 7 and 8 are based on SCR trip outputs. If contact outputs are used, 4 milliseconds must be added to the operating times.

REFERENCES

1. A.R. van C. Warrington, *Protective Relays: their Theory and Practice*, Volume II. London: Chapman & Hall, 1969, chapter 4.

2. T.S. Madhava Rao, *Power System Protection Static Relays*, New York: McGraw-Hill, 1981, chapter 2.

3. S.B. Wilkinson, C.A. Mathews, M.F. Keeney, "Design considerations for a new ground distance relay," *IEEE Trans. on Power Apparatus and Systems*, vol. 98, pp. 1566-1575, 1979.

APPENDIX I

General equation for fault current (including dc offset):

$$i(t) = I\sin(\omega t + \alpha - \theta) - I\sin(\alpha - \theta)e^{-t/T}$$

θ = arctan ($\omega L/R$) = impedance angle of power system $\quad \theta := 85$

$\omega := 2\cdot\pi\cdot 60 \qquad \omega = 376.991$

α = angle on voltage wave at which fault occurs $\quad \alpha := -5$

L/R = T (for power system) $\qquad T := \tan\left[85\cdot\frac{\pi}{180}\right]\cdot\frac{1}{\omega}$

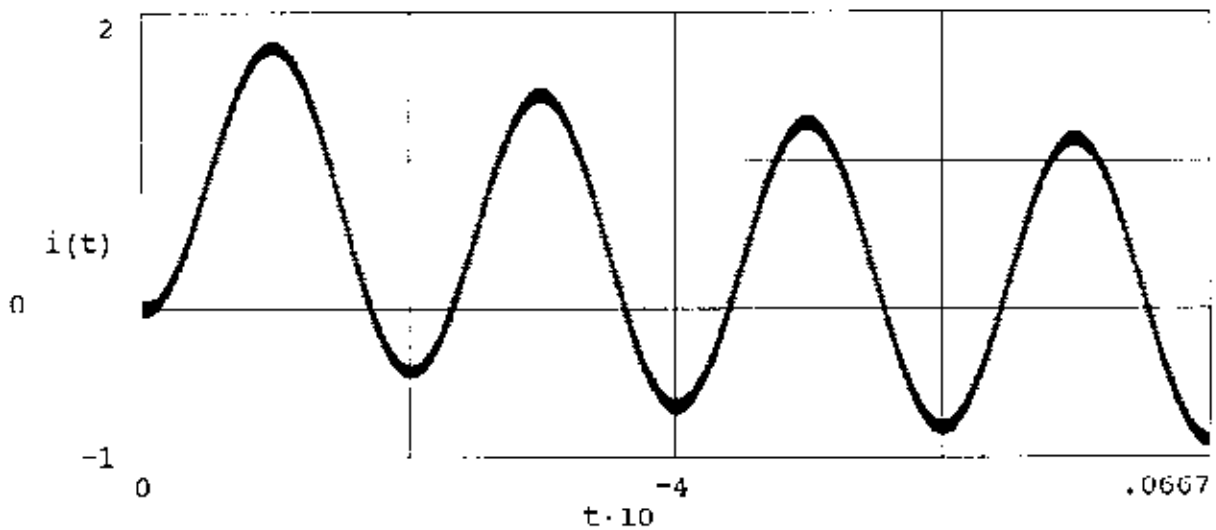
Let: $\phi := (\alpha - \theta)\cdot\frac{\pi}{180} \qquad \phi = -1.571$ radians $\quad (\phi = -90$ degrees)

$t := 0,1\ ..667$

note: ω and 1/T are multiplied by 0.0001 to allow time range above to be expressed in seconds

$\omega := \omega\cdot.0001 \qquad T := T\cdot 10000$
$\omega = 0.038 \qquad T = 303.192$

$$i(t) := \sin(\omega\cdot t + \phi) - \sin(\phi)\cdot e^{-\left|\frac{t}{T}\right|}$$



(SECONDS)

APPENDIX I

Primary Impedance = $R + j\omega L$ Replica Impedance = $R2 + j\omega L2$

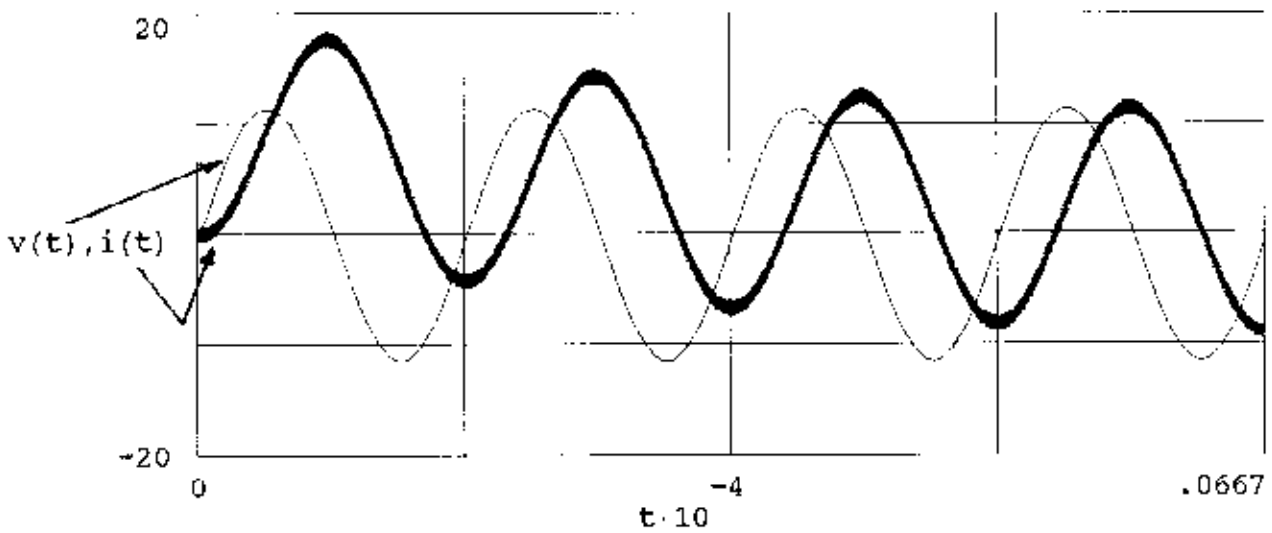
where: $L/R - L2/R2 = T$ $R2 := 1$ $L2 := T$

($\theta 2 = \theta = 85$)

$$v(L) := i(t) \cdot R2 + \frac{d}{dt} i(t) \cdot L2$$

v(t) - voltage across replica impedance

$i(t) := i(t) \cdot 10$ (i(t) is multiplied by 10 to show up better on the plot below)



APPENDIX II

DISCRETE FOURIER TRANSFORM

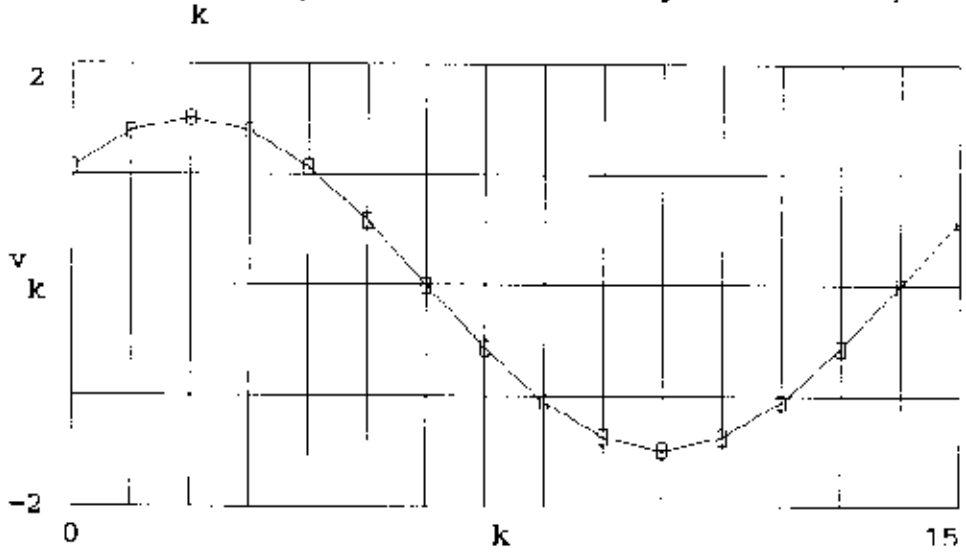
$N := 16 \qquad k := 0,1 \ldots 15$

$$v_k := 1.5 \cdot \sin\left[2 \cdot \pi \cdot \frac{k}{N} + \frac{\pi}{4}\right]$$

(equation to define sampled values)

v_k
1.061
1.386
1.5
1.386
1.061
0.574
0
-0.574
-1.061
-1.386
-1.5
-1.386
-1.061
-0.574
0
0.574

(v_k = sampled values of voltage sine wave)



real (R) and imaginary (I) equations for full-cycle DFT

$$R := \frac{2}{N} \cdot \sum_k v_k \cdot \sin\left[2 \cdot \pi \cdot \frac{k}{N}\right] \qquad I := \frac{2}{N} \cdot \sum_k v_k \cdot \cos\left[2 \cdot \pi \cdot \frac{k}{N}\right]$$

$R = 1.061$ $\qquad I = 1.061$

$$PEAK := \sqrt{R^2 + I^2} \qquad ANGLE := \operatorname{atan}\left(\frac{I}{R}\right) \cdot \frac{180}{\pi}$$

$PEAK = 1.5$ $\qquad ANGLE = 45$

APPENDIX III

FULL-CYCLE DISCRETE FOURIER TRANSFORM

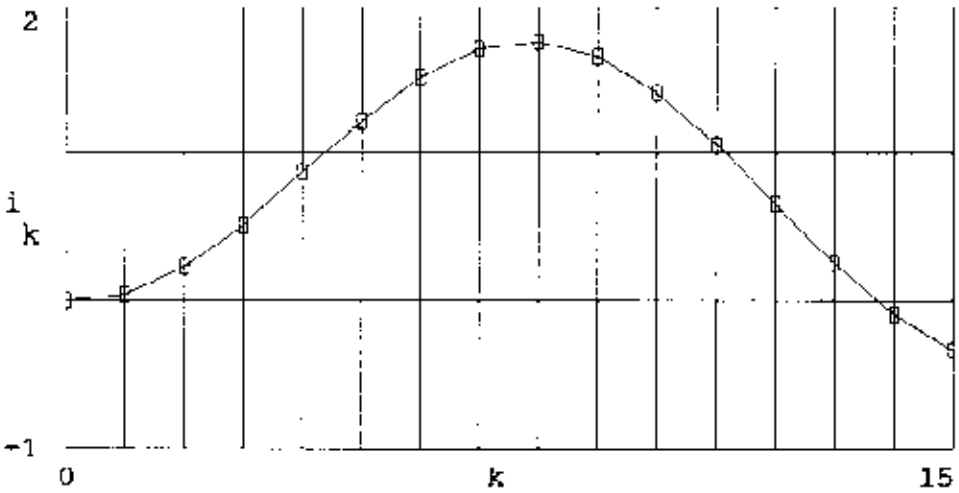
$N := 16$ $k := 0,1 \ldots 15$ $T := 0.0303191$

(1) $$i_k := \sin\left[2\cdot\pi\cdot\left(\frac{k}{N}\right) - 90\cdot\left(\frac{\pi}{180}\right)\right] - \sin\left[-90\cdot\left(\frac{\pi}{180}\right)\right]\cdot e^{-\left[\frac{k}{N\cdot 60\cdot T}\right]}$$

(above equation defines sampled values for current with dc offset - see Appendix I)

i_k
0
0.042
0.226
0.519
0.872
1.225
1.521
1.71
1.76
1.658
1.416
1.068
0.662
0.257
-0.089
-0.327

(i_k = sampled values of current)



real (R) and imaginary (I) equations for DFT:

$$R := \frac{2}{N}\cdot\sum_k i_k\cdot\sin\left[2\cdot\pi\cdot\frac{k}{N}\right] \qquad I := \frac{2}{N}\cdot\sum_k i_k\cdot\cos\left[2\cdot\pi\cdot\frac{k}{N}\right]$$

$R = 0.132$ $I = -0.962$

$$PEAK := \sqrt{R^2 + I^2} \qquad ANGLE := \operatorname{atan}\left[\frac{I}{R}\right]\cdot\frac{180}{\pi}$$

$PEAK = 0.971$ $ANGLE = -82.194$

actual peak = 1.0 actual angle = -85

APPENDIX III

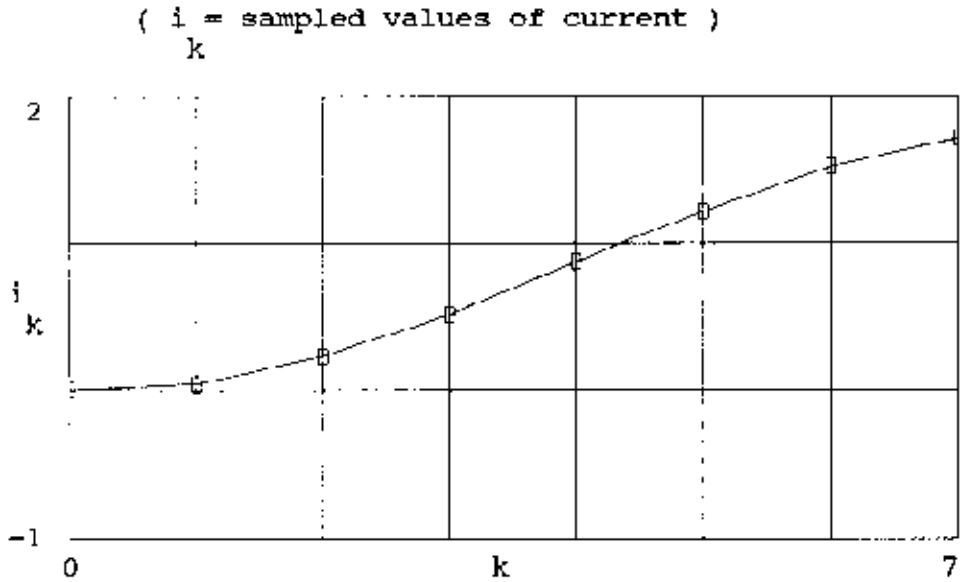
HALF-CYCLE DISCRETE FOURIER TRANSFORM

$N := 8 \qquad k := 0,1 \ldots 7 \qquad T := 0.0303191$

(2) $$i_k := \sin\left(\pi \cdot \frac{k}{N} - 90 \cdot \frac{\pi}{180}\right) - \sin\left(-90 \cdot \frac{\pi}{180}\right) \cdot e^{-\left(\frac{k}{N \cdot 2 \cdot 60 \cdot T}\right)}$$

(above equation defines sampled values for current with dc offset - see Appendix I)

i_k
0
0.042
0.226
0.519
0.872
1.225
1.521
1.71



real (R) and imaginary (I) equations for DFT:

$$R := \frac{2}{N} \cdot \sum_k i_k \cdot \sin\left[\pi \cdot \frac{k}{N}\right] \qquad I := \frac{2}{N} \cdot \sum_k i_k \cdot \cos\left[\pi \cdot \frac{k}{N}\right]$$

$R = 1.097 \qquad I = -0.682$

$$PEAK := \sqrt{R^2 + I^2} \qquad ANGLE := \operatorname{atan}\left[\frac{I}{R}\right] \cdot \frac{180}{\pi}$$

$PEAK = 1.292 \qquad ANGLE = -31.843$

actual peak = 1.0 actual angle = -85